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Primordial Magnetic Fields and Stochastic GW Backgrounds

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Abstract. Operating resonant mass detectors set interesting bounds on diffused backgrounds of gravitational radiation and in the next five years the wide-band interferometers will also look for stochastic sources. In this lecture the interplay among relic GW backgrounds and large scale magnetic fields will be discussed. Magnetic fields may significantly affect the thermal history of the Universe in particular at the epoch electroweak symmetry breaking and shortly after. A review of some old an new results on the spectral properties of stochastic GW backgrounds will be presented. The possible rôle of primordial magnetic fields as a source of gravitational radiation will be outlined. It will be shown that the usual bound on stochastic GW backgrounds coming from the standard big bang nucleosynthesis (BBN) scenario can be significantly relaxed.

1. Introduction

The aim of this lecture is to outline the possible interplay between primordial magnetic fields and relic backgrounds of gravitational radiation. In section 2 the essential features of our magnetized local Universe will be outlined with the aim of supporting the idea that some primordial magnetic field had to exist prior to the decoupling of the radiation from matter. In section 3 the main spectral properties of stochastic GW backgrounds will be reviewed with special emphasis on the present goals of direct experimental search and on the foreseen primordial signals. In section 4 it will be argued that primordial magnetic fields can have an (indirect) impact on the formation of light nuclei. This observation leads to a scenario of BBN which is interesting in its own right: the BBN with matter–antimatter domains. Section 5 contains our concluding remarks.

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2. A magnetized local Universe

The first speculations concerning large scale magnetic fields are contained in a seminal paper of Fermi [1] whose idea was that cosmic rays are in equilibrium not with the sun (or more generically with stars), as proposed by Alfvén [2], but with the whole galaxy. In [1] the Milky Way was viewed as a magnetized gravitationally bound system with a large scale field $\mathcal{O}(\mu\mathrm{G})$. Later this model was further explored in collaboration with S. Chandrasekar [3] trying to connect galactic magnetic field an galactic angular momentum. Today, more than half a century after these pioneering attempts we know that some of the ideas are still valid. For a more complete (but still too short) review on the subject of large scale magnetic fields in cosmology we refer the reader to some recent review written by the author [4] and to some excellent reviews [5, 6, 7, 8, 9] concerning the astronomical sides of this manifold problem. In the following only few important points will be discussed.

Astronomical magnetic fields are certainly strong enough to influence the dynamics of gas in the galaxies and could have been important during the formation of the galaxy. We do know that galaxies are not the only magnetized gravitationally bound systems in the Universe. Clusters of galaxies, the intra-cluster medium and the local super-cluster seems to be all magnetized. The present knowledge is a result of the progress of fifty years of amazing achievements in astronomy. The first attempts of measuring magnetic fields came through optical polarization of starlight assuming that magnetic fields are aligned along the dust grains [12]. The development of radio-astronomy made possible accurate determinations of magnetic fields in the interstellar medium through Faraday Rotation measurements [5, 8]. Polarization of synchrotron emission allowed us to estimate the random component of large scale fields [9]. Observations of x-ray satellites produced more reliable "maps" of relativistic electrons in the intra-cluster medium leading, ultimately, to a rather compelling evidence of a magnetized medium on associated with individual galaxies [13].

2.1. A controversial origin

In spite of the amazing experimental achievements, the origin of large-scale magnetic fields remains still controversial and observational tests that could discriminate between competing theories represent a challenge for existing astronomical facilities. In particular, it seems puzzling that over very different length scales, from galaxies to super-clusters, the magnetic field strength is always $\mathcal{O}(\mu\mathrm{G})$. By itself the μ G field strength may indicate that magnetic fields are, today, all in *equipartition*, i.e. the idea that magnetic and kinetic energy densities may be, after all, comparable. Today, in fact roughly $B^2 \simeq T_{\rm cmb}^4$, where $T_{\rm cmb}$ is the Cosmic Microwave Background (CMB) temperature. The relativistic electron density is sometimes estimated using equipartition. However, equipartition is not an experimental evidence, it is a working hypothesis which may or may not be realized in the system under observation. For instance equipartition probably holds for the Milky Way but it does not seem to be valid in the Magellanic Clouds [10]. The average equipartition field strengths in galaxies ranges from the 4μ G of M33 up to the 19μ G of NGC2276 [11].

As far as galactic magnetic fields are concerned the common lore is that initially small magnetic inhomogeneities (with large correlation scale, of the order of the fraction of the Mpc) are amplified thanks to the global rotation of the galaxy. This is, *in nuce*, the dynamo theory. The dynamo theory has many important aspects (see [14] for a critical review) and it is formulated in the framework of non-relativistic magnetohydrodynamics (MHD). Thanks to the linearity of MHD equations in the mean magnetic fields, some initial conditions for the galactic magnetic field *must* be postulated. This is the so-called initial *seed* hypothesis. Furthermore, by roughly comparing the rotation period of the galaxy with its age we are led to conclude that the maximal achievable amplification one can obtain is of the order of the exponential of the number of rotations performed by the galaxy, i.e. $\mathcal{O}(30)$. During the gravitational collapse of the protogalaxy, the magnetic flux is approximately conserved. Since the physical size of the system shrinks during collapse, then the magnetic field increases.

If this picture would be true, the presently observed magnetic field in galaxies could be the result of the amplification of an initial seed as small as $\mathcal{O}(10^{-23}\mathrm{G})$, coherent over the scale of the gravitational collapse of the protogalaxy. However, numbers have to be much less generous. We assumed *exact* flux freezing during gravitational collapse. This is not always the case. Furthermore, it has been realized that thanks to the dynamo action not only large-scale fields are amplified but also small scale fields increase substantially their amplitude. Eventually the small scale fields may swamp the large-scale dynamo action. According to recent estimates $B_{\mathrm{seed}} \simeq \mathcal{O}(10^{-18}\mathrm{G})$. While the beginning of the dynamo is fixed, essentially, by initial conditions, its end is due to back-reaction effects driving the large-scale magnetic field on its *approximate* equipartition value.

Even if the dynamo would be the correct explanation for the magnetic field of the galaxy, it is still a puzzle why clusters should have magnetic fields of the same strength. Clusters rotate much less than galaxies and dynamo theory would not give a significant increase in the amplitude of a primeval large scale field. Furthermore, experimental evidence tells us that there are magnetic fields in clusters which are not associated with individual galaxies. One can certainly think of some magnetic reconnection mechanism, analogous to the one of flares where the sun ejects magnetic flux together with plasma. In this case galaxies would eject magnetic flux in the intra-cluster medium. This idea does not tell why magnetic fields in the intra-cluster medium are coherent over 100 kpc.

These rather peculiar features of our magnetized Universe seem to indicate that large-scale magnetic fields may have a primordial nature. Even assuming that cluster and supercluster magnetic fields are generated by some different astrophysical mechanism which by chance leads to a $\mathcal{O}(\mu\mathrm{G})$ field, specific initial conditions for the dynamo mechanism should be anyway postulated in order to explain the origin of the galactic magnetic field. It would seem rather easy to get a magnetic field $\mathcal{O}(10^{-18}\mathrm{G})$ at the onset of gravitational collapse. This is not the case. In fact, even if the field has, apparently, small amplitude (if compared, for instance, to the terrestrial magnetic field which is $\mathcal{O}(\mathrm{G})$) its correlation scale must be huge. Notice that a quantum mechanical fluctuation in a box of 1 Mpc is $\mathcal{O}(10^{-60}\mathrm{G})$..

Different possibilities can then be envisaged.

- Large-scale magnetic fields are *not* primordial and are only explained on astrophysical basis. This possibility requires initial conditions for the dynamo evolution. These initial conditions have to be postulated.

- There was some (small) primordial field resulting in a $\mathcal{O}(10^{-18}\mathrm{G})$ at the time of gravitational collapse over the typical scale of the collapse. Then galactic magnetic fields may be generated via dynamo.

- Large-scale magnetic fields are the result of a primordial field $\mathcal{O}(10^{-10}\mathrm{G})$ present at the onset of gravitational collapse. In this case the dynamo action would not even be required and the problem of magnetic fields in clusters will be relaxed, if not completely solved.

Among all these possibilities it is very hard to decide on the basis of *simulations* or on the basis of *theoretical constraints*. A more clever way of discriminating among these ideas is to look at the sky and check if the predictions of the primordial theory are verified in nature. This is difficult but not impossible since the fully primordial theory (without dynamo action) predicts a specific parity of the field with respect to rotations by π about the galactic center which is different from the prediction of the fully primordial hypothesis.

2.2. The primordial hypothesis

The physical reason why magnetic fields, unlike other relics, are preserved by the cosmological evolution is that the Universe was (and partially still is) an extremely good conductor. Consider, for simplicity, a (conformally flat) Friedmann-Robertson-Walker Universe

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2], \tag{1}$$

where η is the conformal time coordinate and $a(\eta)$ is the scale factor which, for instance, evolves linearly during a radiation-dominated phase of expansion, quadratically during matter-domination, hyperbolically during and exact de Sitter phase.

In FRW space-time we can write, for instance, two of the basic laws of resistive MHD, also called, sometimes, Alfvén theorems

$$\frac{d}{d\eta}\int_{\Sigma}\vec{B}\cdot d\vec{\Sigma} = -\frac{1}{\sigma}\int_{\Sigma}\vec{\nabla}\times\vec{\nabla}\times\vec{B}\cdot d\vec{\Sigma}, \tag{2}$$

$$\frac{d}{d\eta}\mathcal{H}_M = -\frac{1}{\sigma}\int_V d^3x\ \vec{B}\cdot\vec{\nabla}\times\vec{B}, \tag{3}$$

where Σ is an arbitrary closed surface which moves with the plasma and where

$$\mathcal{H}_M = \int_V d^3x\vec{A}\ \cdot\vec{B}, \tag{4}$$

is the magnetic helicity and the integration volume V is defined in such a way that $\vec{B}$ is parallel to the surface ∂V which bounds V. In Eqs. (3)–(4) $\vec{A}$ is the vector potential [2] and σ is the conductivity.

In the hot big bang model the temperature increases when we go back in time and so does the conductivity. In the limit of infinite conductivity (sometimes called ideal or superconducting limit) the right hand side of Eqs. (2) and (3) goes to zero. In the ideal limit both the magnetic flux and the magnetic helicity are *exactly* conserved and Eq. (2) implies, that the magnetic flux lines are always glued together with the plasma element,

[2] The quantities appearing in Eqs. (2)–(4) are defined in curved space and they are related to their flat-space counterpart as : $\vec{B} = a^2\vec{\mathcal{B}}$, $\vec{A} = a\vec{\mathcal{A}}$, $\sigma = \sigma_c a$.

or, for short, that magnetic flux is frozen into the plasma element. From Eq. (3), also the magnetic helicity is conserved if $\sigma \to \infty$. This means the number of knots and twists in the magnetic flux lines stays always the same. From the physical point of view, the two Alfvén theorems can be understood in simple terms. Magnetic field lines must be closed (because of transversality). However there could be different topological situations. For instance closed loops may have no intersections. In this case the helicity is zero. There could be however the situation where a loop is twisted (like some type of Moebius stripe) or the case where two loops are connected (like the rings of a chain). In these situations the magnetic helicity is non zero and it is conserved in the superconducting limit. While the conservation of magnetic flux tell us that the *energetical* properties of the magnetic field distribution are conserved, the conservation of the magnetic helicity implies the conservation of the *topological* properties of the magnetic flux lines.

In the early Universe the contribution of the resistivity, i.e. $1/\sigma$ is never zero and, therefore, the conservation of the flux and of the helicity can be only approximate. The quantity

$$r_{\rm B}(L) = \frac{\rho_{\rm B}(L,\eta)}{\rho_{\gamma}(\eta)} \simeq \frac{\langle|\vec{\mathcal{B}}(L,\eta)|^2\rangle}{T^4(\eta)}, \tag{5}$$

is approximately conserved all along the time evolution since, because of of flux freezing $|\vec{\mathcal{B}}| \sim a^{-2}$ and, because of adiabatic evolution $T \sim a^{-1}$. In Eq. (5) L is the typical coherence scale of the field. In terms of $r_{\rm B}$ we can easily write the requirements discussed above in the context of the dynamo mechanism. In particular, if we assume that the amplification of the magnetic field due to dynamo was $\mathcal{O}(30)$ e-folds and that flux was exactly frozen during the collapse of the protogalaxy, we are led to demand, in order to turn on successfully the dynamo action, that $r_{\rm B} \geq \mathcal{O}(10^{-34})$. As previously pointed out, these considerations are rather naive and the realistic requirement is that $r_{\rm B} \geq \mathcal{O}(10^{-24})$ corresponding to a field $\mathcal{O}(10^{-18}{\rm G})$ at the onset of gravitational collapse and over a typical scale $L \sim 1$ Mpc.

If the dynamo mechanism is invoked, the primordial content of the magnetic field sets the initial conditions of the MHD evolution. It could also happen that, since the generated magnetic fields are rather large, there is no need of dynamo amplification and all the amplification occurs during the gravitational collapse. Atypical value of primordial field may be of the order of $r_{\rm B} \sim 10^{-8}$–10^{-9}.

Back in the late sixties Harrison [15] suggested that the initial conditions of the MHD equations might have something to do with cosmology in the same way as it was suggested that the primordial spectrum of gravitational potential fluctuations (i.e. the Harrison-Zeldovich spectrum) might be produced in some primordial phase of the evolution of the Universe. Since then, several mechanisms have been invoked in order to explain the origin of the magnetic seeds and few of them are compatible with inflationary evolution. It is not my purpose to review here all the different mechanisms which have been proposed so far (see, for instance, [4]). The cosmological mechanisms can be *causal* mechanisms (if the magnetic seeds are produced at a given time inside the horizon) and *inflationary* (if correlations in the magnetic field are produced outside the horizon). Both classes of models have their own virtues and their own problems. Causal mechanisms, for instance, lead to large magnetic fields but over small length-scales and, typically, the scale of the relevant domains at the onset of gravitational collapse is much smaller than the Mpc. On the other hand, inflationary mechanisms can efficiently produce large magnetized domains but with very small field intensity.

From the physical point of view it would be desirable to have a model where small quantum mechanical fluctuations of gauge fields are amplified thanks to the dynamical evolution. Then the amplified quantum mechanical fluctuations will become, eventually, the initial conditions of the MHD evolution. This is in full analogy with what it is done with scalar fluctuations of the metric in the context of ordinary inflationary models. The current explanation of the detected CMB anisotropies is indeed that they are the result of amplified fluctuations of the metric. The problem with gauge fields is that their evolution equations are qualitatively very different from the evolution equations of the fluctuations of the metric. Quantum mechanical fluctuations of gauge fields in four space-time dimensions they are not likely to be amplified. This is one of the main motivations, in this context, in order to go beyond four dimensions and study if and how magnetic fields are generated when the gauge coupling is effectively time dependent [17, 18, 19, 20]. In the context of pre-big bang models [16] $r_{\rm B}$ can be as large as 10^{-8} [17, 18]. If the variation of the gauge couplings occurs during a de Sitter stage of expansion we can get $r_{\rm B} \sim 10^{-12}$ [19].

In conclusion we can say that astrophysical mechanisms for the origin of large-scale magnetic fields have to rely on some initial conditions at the epoch of gravitational collapse. Cosmology is able to provide these initial conditions in a number of different models. The situation is, in this sense, not different from what happens in the physics of CMB anisotropies. The common feature of various mechanisms producing magnetic fields in the early Universe is that magnetic fields are generated not only over the typical scale of the gravitational collapse of the protogalaxy but also over different physical scales. This means that the magnetic fields produced in the far past and giving, for instance, initial conditions for the dynamo mechanism, will also influence other moments of the thermodynamical history of the Universe. Two examples are the moment of electroweak symmetry breaking and the moment of BBN.

2.3. Hypermagnetic knots and electroweak symmetry breaking

Since a generic magnetic field configuration at finite conductivity leads to an energy-momentum tensor which is anisotropic and which has non-vanishing transverse and traceless component (TT), if magnetic fields are present inside the horizon at some epoch they can radiate gravitationally. In more formal terms this statement can be understood since the TT components of the energy momentum tensor acts as a source term for the TT fluctuations of the geometry which are associated with gravitational waves. A non-trivial example of this effect is provided by magnetic knot configurations [21] which are transverse (magnetic) field configurations with a topologically non-trivial structure in the flux lines.

For sufficiently high temperatures and for sufficiently large values of the various fermionic charges the $SU(2)_L \otimes U(1)_Y$ symmetry is restored and non-screened vector modes will now correspond to the hypercharge group. Topologically non-trivial configurations of the hypermagnetic field ($\vec{\mathcal{H}}_Y$) can be related to the baryon asymmetry of the Universe (BAU) [22, 23, 24, 25] and they can also radiate gravitationally [26, 27]. In this context the value of the BAU is directly related to the amplitude of the stochastic GW background. The evolution equations of the hypercharge field at finite conductivity imply that the largest modes which can survive in the plasma are the ones associated with the hypermagnetic conductivity frequency which is roughly eight orders of magnitude smaller than the temperature at the time of the electroweak phase transition which I take to occur around 100 GeV.

If a hypermagnetic background is present for $T > T_c$, then the energy momentum tensor will acquire a small anisotropic component which will source the evolution equation of the tensor fluctuations $h_{\mu\nu}$ of the metric $g_{\mu\nu}$:

$$h_{ij}'' + 2\mathcal{H}h_{ij}' - \nabla^2 h_{ij} = -16\pi G \tau_{ij}^{(T)}. \tag{6}$$

where $\tau_{ij}^{(T)}$ is the *tensor* component of the *energy-momentum tensor* of the hypermagnetic fields. Suppose now that $|\vec{\mathcal{H}}|$ has constant amplitude and that it is also homogeneous. Then we can easily deduce the critical fraction of energy density present today in relic gravitons of EW origin

$$\Omega_{\rm gw}(t_0) = \frac{\rho_{\rm gw}}{\rho_c} \simeq z_{\rm eq}^{-1} r^2, \quad \rho_c(T_c) \simeq N_{\rm eff} T_c^4 \tag{7}$$

($z_{\rm eq}$ is the redshift from the time of matter-radiation, equality and$N_{\rm eff} = 106.75$ is the effective number of spin degrees of freedom at $T_c \sim 100$ GeV). Because of the structure of the equations describing the evolution of the system at finite fermionic density and finite conductivity [24, 25], stable hypermagnetic fields will be present not only for $\omega_{\rm ew} \sim k_{\rm ew}/a$ but for all the range $\omega_{\rm ew} < \omega < \omega_\sigma$ where ω_σ is the diffusivity frequency. The (present) values of $\omega_{\rm ew}$ is

$$\omega_{\rm ew}(t_0) \simeq 2.01 \times 10^{-7} \left(\frac{T_c}{1\,{\rm GeV}}\right) \left(\frac{N_{\rm eff}}{100}\right)^{1/6} {\rm Hz}. \tag{8}$$

Thus, $\omega_\sigma(t_0) \sim 10^8 \omega_{\rm ew}$. Suppose now that $T_c \sim 100$ GeV; than we will have that $\omega_{\rm ew}(t_0) \sim 10^{-5}$ Hz. Suppose now that

$$|\vec{\mathcal{H}}|/T_c^2 \geq 0.3. \tag{9}$$

This requirement imposes $r \simeq 0.1$–0.001 and, consequently,

$$h_0^2 \Omega_{\rm GW} \simeq 10^{-7} - 10^{-8}. \tag{10}$$

Notice that this signal would occurr in a (present) frequency range between 10^{-5} and 10^3 Hz. This signal satisfies the presently available phenomenological bounds on the graviton backgrounds of primordial origin (see the following section). The pulsar timing bound is automatically satisfied since our hypermagnetic background is defined for $10^{-5}{\rm Hz} \leq \omega \leq 10^3{\rm Hz}$. The large-scale bounds would imply $h_0^2\Omega_{\rm GW} < 7 \times 10^{-11}$ but a at much lower frequency (i.e. 10^{-18} Hz). The signal discussed here is completely absent for frequencies $\omega < \omega_{\rm ew}$. Notice that this signal is clearly distinguishable from other stochastic backgrounds occurring at much higher frequencies (GHz region) like the ones predicted by quintessential inflation and pre-big bang cosmology (see following Section). The frequency of operation of the interferometric devices (VIRGO/LIGO) is located between few Hz and 10 kHz. The frequency of operation of LISA is well below the Hz (i.e. 10^{-3}Hz, approximately). In this model the signal can be located both in the LISA window and in the VIRGO/LIGO window due to the hierarchy between the hypermagnetic diffusivity scale and the horizon scale at the phase transition [23, 25].

3. Spectral Properties of stochastic GW backgrounds

The fraction of critical energy density ρ_c stored in relic gravitons at the present (conformal) time η_0 per each logarithmic interval of the physical frequency f

$$\Omega_{\rm GW}(f,\eta_0) = \frac{1}{\rho_c}\frac{{\rm d}\rho_{\rm GW}}{{\rm d}\ln f} = \overline{\Omega}(\eta_0)\,\omega(f,\eta_0) \tag{11}$$

is the quantity we will be mostly interested in.

The frequency dependence in $\Omega_{\rm GW}(f,\eta_0)$ is a specific feature of the mechanism responsible for the production of the gravitons and, in a given interval of the present frequency, the slope of the logarithmic energy spectrum can be defined as

$$\alpha = \frac{d\,\ln\omega(f,\eta_0)}{d\,\ln f}. \tag{12}$$

If, in a given logarithmic interval of frequency, $\alpha < 0$ the spectrum is *red* since its dominant energetical content is stored in the infra-red. If, on the other hand $0 < \alpha \leq 1$ the spectrum is *blue*, namely a mildly increasing logarithmic energy density. Finally if $\alpha > 1$ we will talk about *violet* spectrum whose dominant energetical content is stored in the ultra-violet. The case $\alpha = 0$ corresponds to the case of scale-invariant (Harrison-Zeldovich) logarithmic energy spectrum.

Every sudden variation of the background geometry from one regime of expansion to the other leads inevitably to the production of graviton pairs which are stochastically distributed [28, 29, 30]. Valuable reviews on the subject are given in Refs. [31, 32, 33]. The amplitude of the detectable signal depends, however, upon the specific model of curvature evolution. In ordinary inflationary models the amount of gravitons produced by a variation of the geometry is notoriously quite small. This feature can be traced back to the fact that $\Omega_{\rm GW}(f,\eta_0)$ is either a decreasing or (at most) a flat function of the present frequency. Suppose, for simplicity, that the ordinary inflationary phase is suddenly followed by a radiation dominated phase turning, after some time, into a matter dominated stage of expansion. The logarithmic energy spectrum will have, as a function of the present frequency, two main branches : an infra-red branch (roughly ranging between 10^{-18} Hz and 10^{-16} Hz) and a flat (or possibly decreasing) branch between 10^{-16} and 100 MHz.

The flat branch of the spectrum is mainly due to those modes leaving the horizon during the inflationary phase and re-entering during the radiation dominated epoch. The infra-red branch of the spectrum is produced by modes leaving the horizon during the inflationary phase and re-entering during the matter dominated epoch.

Starting from infra-red we have that the COBE observations of the first thirty multipole moments of the temperature fluctuations in the microwave sky imply that the GW contribution to the Sachs-Wolfe integral cannot exceed the amount of anisotropy directly detected. This implies that for frequencies f_0 approximately comparable with H_0 and 20 H_0 (where H_0 is the present value of the Hubble constant including its indetermination h_0) $h_0^2\,\Omega_{\rm GW}(f_0,\eta_0) < 7\times 10^{-9}$. Moving toward the ultra-violet, the very small size of the fractional timing error in the arrivals of the millisecond pulsar's pulses requires that $\Omega_{\rm GW}(f_P,\eta_0) < 10^{-8}$ for a typical frequency roughly comparable with the inverse of the observation time during which the pulses have been monitored, i.e. $f_P \sim 10$ nHz.

Finally, if we believe the simplest (homogeneous and isotropic) big-bang nucleosynthesis (BBN) scenario we have to require that the total fraction of critical energy density stored in relic gravitons at the BBN time does not exceed the energy density stored in relativistic matter at the same epoch. Defining $\Omega_{\gamma}(\eta_0)$ as the fraction of critical energy density presently stored in radiation we have that the BBN consistency requirement demands

$$h_0^2 \int_{f_{\rm ns}}^{f_{\rm max}} \Omega_{\rm GW}(f,\eta_0)\, {\rm d}\ln f \;\leq\; 5\,\times\,10^{-6}\,\Delta N_{\rm eff} \tag{13}$$

where $f_{\rm ns} \simeq 0.1$ nHz is the present value of the frequency corresponding to the horizon at the nucleosynthesis time; $f_{\rm max}$ stands for the maximal frequency of the spectrum and it depends upon the specific theoretical model (in the case of ordinary inflationary models $f_{\rm max} = 100$ MHz). In Eq. (13), $\Delta N_{\rm eff}$ is the excess in the effective number of neutrino species which will be discussed in section 4. The constraint expressed in Eq. (13) is *global* in the sense that it bounds the *integral* of the logarithmic energy spectrum. The constraints coming from pulsar's timing errors and from the integrated Sachs-Wolfe effect are instead *local* in the sense that they bound the value of the logarithmic energy spectrum in a specific interval of frequencies.

In the case of stochastic GW backgrounds of inflationary origin, owing to the red nature of the logarithmic energy spectrum, the most significant constraints are the ones present in the soft region of the spectrum, more specifically, the ones connected with the Sachs-Wolfe effect. Taking into account the specific frequency behavior in the infra-red branch of the spectrum and assuming perfect scale invariance we have that $h_0^2\ \Omega_{\rm GW}(f,\eta_0) < 10^{-15}$ for frequencies $f > 10^{-16}$ Hz. We have to conclude that the inflationary spectra are invisible by pairs of interferometric detectors operating in a window ranging approximately between few Hz and 10 kHz.

In order to illustrate more quantitatively this point we remind the expression of the signal-to-noise ratio (SNR) in the context of optimal processing required for the detection of stochastic backgrounds [34, 35, 36, 37]. By assuming that the intrinsic noises of the detectors are stationary, Gaussian, uncorrelated, much larger in amplitude than the gravitational strain, and statistically independent on the strain itself, one has:

$$\mathrm{SNR}^2 \;=\; \frac{3H_0^2}{2\sqrt{2}\,\pi^2}\,F\,\sqrt{T}\,\left\{\int_0^{\infty} {\rm d}f\,\frac{\gamma^2(f)\,\Omega_{\rm GW}^2(f)}{f^6\,S_n^{(1)}(f)\,S_n^{(2)}(f)}\right\}^{1/2}\;, \tag{14}$$

(F depends upon the geometry of the two detectors and in the case of the correlation between two interferometers $F = 2/5$; T is the observation time). In Eq. (14), $S_n^{(k)}(f)$ is the (one-sided) noise power spectrum (NPS) of the k-th ($k = 1, 2$) detector. The NPS contains the important informations concerning the noise sources (in broad terms seismic, thermal and shot noises) while $\gamma(f)$ is the overlap reduction function which is determined by the relative locations and orientations of the two detectors. Without going through the technical details [38, 39, 40] from the expression of the SNR we want to notice that the achievable sensitivity of a pair of wide band interferometers crucially depends upon the spectral slope of the theoretical energy spectrum in the operating window of the detectors. So, a flat spectrum will lead to an experimental sensitivity which might not be similar to the sensitivity achievable in the case of a blue or violet spectra [38, 41, 42]. In the case of an exactly scale invariant spectrum the correlation of the two (coaligned) LIGO detectors with central corner stations in Livingston (Lousiana) and in Hanford (Washington) will have a sensitivity to a flat spectrum which is $h_0^2\ \Omega_{\rm GW}(100\ {\rm Hz}) \simeq$

6.5×10^{-11} after one year of observation and with signal-to-noise ratio equal to one [38]. This implies that ordinary inflationary spectra are (and will be) invisible by wide band detectors since the inflationary prediction, in the most favorable case (i.e. scale invariant spectra), undershoots the experimental sensitivity by more than four orders of magnitude.

3.1. Scaling violations in graviton spectra

In order to have a large detectable signal between 1 Hz and 10 kHz we have to look for models exhibiting scaling violations for frequencies larger than the mHz. The scaling violations should go in the direction of blue ($0 < \alpha \leq 1$) or violet ($\alpha > 1$) logarithmic energy spectra. Only in this case we shall have the hope that the signal will be large enough in the window of wide band detectors. Notice that the growth of the spectra should not necessarily be monotonic: we might have a blue or violet spectrum for a limited interval of frequencies with a spike or a hump.

Suppose now, as a toy example, that the ordinary inflationary phase is not immediately followed by a radiation dominated phase but by a quite long phase expanding slower than radiation [43]. This speculation is theoretically plausible since we ignore what was the thermodynamical history of the Universe prior to BBN. If the Universe expanded slower than radiation the equation of state of the effective sources driving the geometry had to be, for some time, stiffer than radiation. This means that the effective speed of sound c_s had to lie in the range $1/\sqrt{3} < c_s \leq 1$. Then the resulting logarithmic energy spectrum, for the modes leaving the horizon during the inflationary phase and re-entering during the stiff phase, is tilted toward large frequencies with typical (blue) slope given by [43]

$$\alpha = \frac{6c_s^2 - 2}{3c_s^2 + 1}, \quad 0 < \alpha \leq 1. \tag{15}$$

A situation very similar to the one we just described occurs in quintessential inflationary models [44]. In this case the tilt is maximal (i.e., $\alpha = 1$) and a more precise calculation shows the appearance of logarithmic corrections in the logarithmic energy spectrum which becomes [41, 42, 43, 44] $\omega(f) \propto f \ln^2 f$. The maximal frequency $f_{\rm max}(\eta_0)$ is of the order of 100 GHz (to be compared with the 100 MHz of ordinary inflationary models) and it corresponds to the typical frequency of a spike in the GW background. In quintessential inflationary models the relic graviton background will then have the usual infra-red and flat branches supplemented, at high frequencies (larger than the mHz and smaller than the GHz) by a true hard branch [41, 42] whose peak can be, in terms of $h_0^2\ \Omega_{\rm GW}$, of the order of 10^{-6}, compatible with the BBN bound and roughly eight orders of magnitude larger than the signal provided by ordinary inflationary models.

An interesting aspect of this class of models is that the maximal signal occurs in a frequency region between the MHz and the GHz. Microwave cavities can be used as GW detectors precisely in the mentioned frequency range [45]. There were published results reporting the construction of this type of detectors [46] and the possibility of further improvements in the sensitivity received recently attention [47]. Our signal is certainly a candidate for this type of devices.

3.2. String cosmological models

In string cosmological models [16] of pre-big bang type $h_0^2\ \Omega_{\rm GW}$ can be as large as 10^{-7}–10^{-6} for frequencies ranging between 1 Hz and 100 GHz [48, 49, 50, 51]. In these types of models the logarithmic energy spectrum can be either blue or violet depending upon the given mode of the spectrum. If the mode under consideration left the horizon during the dilaton-dominated epoch the typical slope will be violet (i.e. $\alpha \sim 3$ up to logarithmic corrections). If the given mode left the horizon during the stringy phase the slope can be also blue with typical spectral slope $\alpha \sim 6 - 2(\ln g_1/g_s/\ln z_s)$ where g_1 and g_s are the values of the dilaton coupling at the end of the stringy phase and at the end of the dilaton dominated phase; z_s parametrizes the duration of the stringy phase. This behaviour is representative of the minimal string cosmological scenarios. However, in the non-minimal case the spectra can also be non monotonic. Recently the sensitivity of a pair of VIRGO detectors to string cosmological gravitons was specifically analyzed [24] with the conclusion that a VIRGO pair, in its upgraded stage, will certainly be able to probe wide regions of the parameter space of these models. If we maximize the overlap between the two detectors [24] or if we would reduce (selectively) the pendulum and pendulum's internal modes contribution to the thermal noise of the instruments [25], the visible region (after one year of observation and with SNR equal to one) of the parameter space will get even larger. Unfortunately, as in the case of the advanced LIGO detectors, also in the case of the advanced VIRGO detector the sensitivity to a flat spectrum will be irrelevant for ordinary inflationary models.

4. Relaxing the BBN bound

The strongest constraint on additional energy density in the universe with a radiation-like equation of state is provided by big bang nucleosynthesis (BBN). The additional energy density speeds up the expansion and cooling of the universe, and, consequently, the typical time scale of BBN is reduced in comparison with the standard case. The additional radiation-like energy density may be attributed to some extra relativistic species whose statistics may be either bosonic or fermionic. Since the supplementary species may be fermionic, they have been customarily parametrized in terms of the effective number of neutrino species

$$N_{\rm eff} = 3 + \Delta N_{\rm eff}, \tag{16}$$

where $\Delta N_{\rm eff} = 0$ corresponds to the standard case with no extra energy density. The standard BBN (SBBN) results are in agreement with the observed abundances for $N_{\rm eff} =$ 2–4, giving thus an upper limit $\Delta N_{\rm eff} \leq 1$.

If hypermagnetic fields are present at the electroweak time, matter–antimatter domains can be generated and persist until the time of BBN [23, 24]. This possibility is rather interesting since matter–antimatter domains suggest a slightly different scenario of BBN which has been developed independently on the motivation stemming from hypermagnetic fields [52, 53, 54, 55, 57, 58].

Provided that matter–antimatter domains are present at the onset of big bang nucleosynthesis (BBN), the number of allowed additional relativistic species increases, compared to the standard scenario when matter–antimatter domains are absent [58].

The extra relativistic species may take the form of massless fermions or even massless bosons, like relic gravitons. The number of additional degrees of freedom compatible with BBN depends, in this framework, upon the typical scale of the domains and the antimatter fraction. Since the presence of matter–antimatter domains allows a reduction of the neutron to proton ratio prior to the formation of ^{4}He, large amounts of radiation-like energy density are allowed. The present critical fraction of energy density stored in relic gravitons, i.e. (13) depends upon $\Delta N_{\rm eff}$ whose range of variation can be translated into constraints on the energy density of relic gravitational waves produced prior to BBN.

Various resonant mass detectors are now operating [59, 60, 61, 62]. In [63], the first experiment of cross-correlation between two cryogenic detectors has been reported with the purpose of giving an upper limit on $h_0^2\Omega_{\rm GW}$. The two detectors are Explorer [61] (operating in CERN, Geneva) and Nautilus [62] (operating in Frascati, near Rome). Previous experiments giving upper limits on $h_0^2\Omega_{\rm GW}$ used room temperature detectors. The Rome group obtained then an upper limit $h_0^2\Omega_{\rm GW} < 60$ at a frequency of roughly 905 Hz. The limit is a result of cross-correlation between the two detectors (located at a distance of approximately 600 km) for an integration time of approximately 12 hours. This limit is not competitive with the BBN bound (and also above the critical density bound implying that $\Omega_{\rm GW} < 1$). However, by increasing the correlation time from few hours to few months it is not unreasonable to go below one in $h_0^2\Omega_{\rm GW}$.

Hollow spherical detectors have been recently investigated [64] as a possible tool for the analysis of the relic gravitational wave background. The sensitivity of two correlated spherical detectors could be $\mathcal{O}(10^{-6})$ in $h_0^2\Omega_{\rm GW}$ for the frequency of resonance which lies between 200 and 400 Hz. In this case the ABBN bound and the experiment will be certainly competitive. Dual spherical detectors [65] may reach a sensitivity, in $h_0^2\Omega_{\rm GW}$, which is again $\mathcal{O}(10^{-6})$ in the kHz region.

Wide-band interferometers [66, 67, 68, 69], a promising tool not yet available but close to the phase of preliminary run, will also be able to probe stochastic sources.

The observation we want to make here is very simple. Consider, for instance, the situation where a stochastic background is detected. The bound of Eq. (13) can help in deciding if the source is cosmological or not. If the background is cosmological then the bound (13) will be satisfied. If matter–antimatter domain are present at the onset of BBN (a situation not impossible if hypermagnetic fields are evolving at the electroweak epoch) then $\Delta N_{\rm eff}$ may be rather large. As a consequence, provided $h_0^2\Omega_{\rm GW} \leq \mathcal{O}(10^{-4})$, the signal may still be of primordial origin. A significant improvement if compared to the case where $\Delta N_{\rm eff} \sim \mathcal{O}(1)$ where $h^2\Omega_{\rm GW} \leq \mathcal{O}(10^{-6})$.

5. Concluding remarks

CMB experiments are the present of experimental cosmology, GW represent a foreseeable future. The GW spectrum ranges over thirty decades in frequency. GW with (present) frequencies around $f_0 \sim 10^{-18}$ Hz correspond to a wave-length as large as the present Hubble radius. For these waves ideal detectors would be CMB experiments. Between few Hz and 10 kHz is located the operating window of ground based interferometers. The band of resonant mass detectors is around the kHz. Finally between few MHz and few GHz microwave cavities can be used as GW detectors.

Between 10^{-18} Hz and 10 kHz there are, roughly, 22 decades in frequency. The very same frequency gap, if applied to the well known electromagnetic spectrum, would drive us from low-frequency radio waves up to x-rays or γ-rays. As the physics explored by radio waves is very different from the physics probed by γ rays it can be argued that the informations carried by low and high frequency GW must derive from two different physical regimes of the theory.

In particular, low frequency GW are sensitive to the large scale features of the given cosmological model and of the underlying theory of gravity, whereas high frequency GW are sensitive to the small scale features of a given cosmological model and of the underlying theory of gravity. For instance string theory is expected to lead to a description of gravity which resembles very much Einstein-Hilbert gravity at large scales but which can deviate from Einstein-Hilbert gravity at smaller scales. That is only one of the many reasons why it is very important to have GW detectors operating over different frequency bands.

An apparently unrelated problem is the controversial origin of our magnetized Universe. The primordial hypothesis is certainly viable. Furthermore, astrophysical explanations demand, anyway, some specific tuning whose origin may find explanations in cosmology. Among other interesting signatures, stochastic GW backgrounds could tell us something on the nature and evolution of magnetic fields during the thermodynamical history of the Universe. Few examples in this direction have been provided.

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